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A QUALITATIVE ANALYSIS ON NONCONSTANT GRAININESS OF THE ADAPTIVE GRIDS VIA TIME SCALES

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ABSTRACT. Calculus on time scales plays a crucial role in unifying the continuous and discrete calculus. In this paper, we apply the time scales calculus methods to study qualitatively properties of the numerical solution of second order ordinary differential equations via different finite difference schemes. The properties become particularly interesting in the case when the computational grids are nonuniform, on which the finite difference operators do not commute. To investigate the solution properties, we introduce the graininess function, and express the numerical solution as functions of the variable grid steps, that is, functions of the graininess and its dynamic derivatives implemented by using the time scales analysis. It is found in the study that a linear combination of the consecutive numerical solutions following the pattern of the nonuniform grid used may improve the accuracy of the numerical solution. We validate our results with several constructive computational experiments.

1. Introduction. The study of analysis on time scales was introduced by Hilger in his Ph.D. dissertation [**6**]. The original motivation of the study was to unify continuous analysis and discrete analysis. A significant amount of time scales related publications can be found nowadays, and some of them had proposed interesting applications of the theory and methods. Recently Eloe et al. [**4, 5**] have initiated the application of the calculus on time scales to questions in adaptive, or variable step, computations. The premise presented here is that a collection of calculus rules, valid in both continuous analysis and discrete analysis, will provide new insight into the qualitative properties of numerical solutions obtained via variable step finite difference schemes.

For the sake of exposition, we address a specific boundary value problem (BVP) for a second order ordinary differential equation of

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