ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 36, Number 1, 2006

ON THE GOLDBACH CONJECTURE IN ARITHMETIC PROGRESSIONS

CLAUS BAUER AND WANG YONGHUI

ABSTRACT. It is proved that for a given integer N and for all but $\ll (\log N)^B$ prime numbers $k \le N^{5/48-\varepsilon}$ the following is true: For any positive integers b_i, $i \in \{1,2,3\}$, $(b_i,k)=1$ that satisfy $N \equiv b_1+b_2+b_3 \pmod k$, N can be written as $N=p_1+p_2+p_3$, where the p_i, $i \in \{1,2,3\}$ are prime numbers that satisfy $p_i \equiv b_i \pmod k$.

1. Introduction. Vinogradov [**17**] has proved that every sufficiently large odd positive integer can be written as the sum of three primes. This theorem has been generalized in many ways. In 1953, Ayoub [**1**] proved the following result: *If k is a fixed positive integer, b_i, $i=1,2,3$, are integers with $(b_i,k)=1$ and $J(N;k,b_1,b_2,b_3)$ is the number of solutions of the equation*

$$\begin{cases} N = p_1+p_2+p_3, \\ p_j \equiv b_j \pmod k, \end{cases}$$

then

$$J(N;k,b_1,b_2,b_3) = (N;k)\,\frac{N^2}{2\log^3 N}\,(1+o(1)),$$

where for odd integer $N \equiv b_1+b_2+b_3 \pmod k$,

$$\sigma(N,k) = \frac{C(k)}{k^2}\prod_{p\mid k}\frac{p^3}{(p-1)^3+1}\prod_{\substack{p\mid N\\ p\nmid k}}\frac{(p-1)((p-1)^2-1)}{(p-1)^3+1} \times \prod_{p>2}\left(1+\frac{1}{(p-1)^3}\right),$$

2000 AMS *Mathematics Subject Classification.* Primary 11P32, 11L07.

The work of the second author was supported partly by Chinese NSF Tianyuan Youth Foundation (10126001) and was done while he was a visiting scholar at the Morningside Center.
Received by the editors on January 6, 2003, and in revised form on July 22, 2004.