ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 36, Number 1, 2006

BOUNDED SOLUTIONS OF THIRD ORDER NONLINEAR DIFFERENCE EQUATIONS

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ABSTRACT. We consider the nonlinear difference equation

$$\Delta\Big(a_n\Delta(b_n\Delta x_n)\Big) = q_n f(x_{n+2}), \quad n \in N,$$

where $\{a_n\}$, $\{b_n\}$, $\{q_n\}$ are positive real sequences, f is a real function with $xf(x) > 0$ for all $x \neq 0$. We obtain sufficient conditions for the boundedness of all nonoscillatory solutions of the above equation. Some examples are also given.

1. Introduction. Consider the third order difference equation

$$\text{(E)} \qquad \Delta\Big(a_n\Delta(b_n\Delta x_n)\Big) = q_n f(x_{n+2}), \quad n = 1, 2, \dots$$

where Δ is the forward difference operator defined by $\Delta x_n = x_{n+1} - x_n$, $\{a_n\}$, $\{b_n\}$, $\{q_n\}$ are sequences of positive real numbers, $f : R \to R$ is a real function with $xf(x) > 0$ for $x \neq 0$.

The following convention is used:

$$\sum_{i=k}^{k-t} a_i := 0 \quad \text{for any} \quad k,\, t \in N.$$

By a solution of equation (E) we mean a real sequence $\{x_n\}$, which satisfies equation (E) for all sufficiently large n and is not eventually identically zero. A solution of equation (E) is called nonoscillatory, if it is eventually positive or eventually negative. Otherwise it is called oscillatory. A sequence $\{x_n\}$ is called quickly oscillatory if and only if $x_n = (-1)^n z_n$ for all $n \in N$, where $\{z_n\}$ is a sequence of positive numbers or a sequence of negative numbers.

In recent years there has been an increasing interest in the study of the qualitative behavior of solutions of difference equations. In

Received by the editors on May 1, 2003.
Key words and phrases. Bounded solutions, oscillatory solutions, nonoscillatory solutions, quickly oscillatory solutions.