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WEAKLY KRULL AND RELATED DOMAINS OF THE FORM $D+M$, $A+XB[X]$ AND $A+X^2B[X]$

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ABSTRACT. Let $T = K + M$ and $R = D + M$ be integral domains, where K is a field, M is a nonzero maximal ideal of T, and D is a proper subring of K. We show that R is a weakly Krull domain, respectively, WFD, AWFD, GWFD, if and only if $\operatorname{ht} M = 1$, D is a field, and T is a weakly Krull domain, respectively, WFD, AWFD, GWFD. Let $A \subsetneq B$ be an extension of integral domains, $R = A + XB[X]$, and $D = A + X^2B[X]$. We also show that R is a weakly Krull domain if and only if D is a weakly Krull domain, if and only if $B_{A-\{0\}}$ is a field, $qf(A) \cap B = A$, and $B[X]$ is a weakly Krull domain; that R is a WFD, respectively AWFD, if and only if $qf(A) \cap B = A$, $B[X]$ is a WFD, respectively AWFD, and for each $0 \neq b \in B$, there is a unit u of B such that $ub \in A$ (respectively, an integer $n = n(b) \geq 1$ and a unit u of B such that $ub^n \in A$); and that if $\operatorname{char} B \neq 0$, then R is an AWFD if and only if D is an AWFD.

1. Introduction. In this paper, we determine when three different pullback constructions yield weakly Krull domains, weakly factorial domains, or almost weakly factorial domains. The first two pullbacks we consider are the well known $D + M$ and $A + XB[X]$ constructions, and the third concerns domains of the form $A + X^2B[X]$.

Let R be an integral domain. Then R is called a *weakly Krull domain* if $R = \cap_{P \in X^1(R)} R_P$, where $X^1(R)$ is the set of height-one prime ideals of R, and R has finite character. Examples of weakly Krull domains include Krull domains, one-dimensional Noetherian domains, and one-dimensional semi-quasi-local domains. It is well known that if R is weakly Krull, then $t\text{-}\dim R = 1$ [**9**, Lemma 2.1], that is, every prime t-ideal of R has height-one. A nonzero element a of R is said to be *primary* if aR is a primary ideal of R. As in [**8**], we will call R a *weakly factorial domain* (WFD) if each nonzero nonunit of R is a product of

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