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# REPRESENTATION RESULTS FOR OPERATORS GENERATED BY A QUASI-DIFFERENTIAL MULTI-INTERVAL SYSTEM IN A HILBERT DIRECT SUM SPACE

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ABSTRACT. We study the spectral structure of operators generated as direct sums of self-adjoint extensions of quasi-differential minimal operators on a multi-interval set (self-adjoint vector-operators). Special attention is given to the ordered spectral representation for such operators.

## 1. Introduction.

1.1 *Problem overview.* The modern theory of quasi-differential v-operators originates from the fundamental work of Gesztesy and Kirsch [**10**], where these authors considered a Schrödinger operator generated by the Hamiltonian

$$H = -\frac{d^2}{dx^2} + \left(s^2 - \frac{1}{4}\right)\frac{1}{\cos^2 x}, \quad s > 0. \tag{1}$$

It is clear that the potential in (1) has a countable number of singularities on $\mathbf{R}$, leading to spoiling of the local integrability. In order to overcome this difficulty, operators $T_i$ are constructed, generated by the same Hamiltonian (1) in the coordinate spaces

$$L^2\left(-\frac{\pi}{2} + i\pi,\ \frac{\pi}{2} + i\pi\right),$$

$i \in \mathbf{Z}$, and then the direct sum operator $\oplus_{i\in\mathbf{Z}} T_i$ is considered in the space

$$\bigoplus_{i\in\mathbf{Z}} L^2\left(-\frac{\pi}{2} + i\pi,\ \frac{\pi}{2} + i\pi\right).$$

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