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GENERALIZED VERSION OF THE CHARACTERISTIC NUMBER OF TWO SIMULTANEOUS PELL'S EQUATIONS

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ABSTRACT. Let D be a given square-free natural number and N a given non-zero integer. Extending the earlier work of Mohanty and Ramasamy, we present in this paper a generalized version of the characteristic number of the simultaneous Pell's equations $U^2 - DV^2 = N$ and $Z^2 - gV^2 = h$ where g and h are given integers. A numerical example is provided at the end explaining the application of the method developed in this paper. It is shown that the only positive integral solutions common to the two Pell's equations $U^2 - 11V^2 = 5$ and $Z^2 - 17V^2 = -32$ are $U = 7$, $V = 2$ and $Z = 6$.

1. Introduction. Quite recently, Pell's equation with restriction has been studied by several authors like Anglin, Baker, Davenport, Cohn, Mohanty, Ramasamy, Pinch, Ponnudurai, Tzanakis, etc. An elaborate list of references on this subject has been furnished by Tzanakis [**10, 11**]. Baker and Davenport [**3**] determined the common solutions of the equations $3x^2 - 2 = y^2$ and $8x^2 - 7 = z^2$ by the method of linear forms in logarithms of algebraic numbers. Anglin [**1**] presented a method for solving a system of Pell's equations with the parameters absolutely less than 1000 and, in [**2**], he considered the system $x^2 - Ry^2 = 1$ and $z^2 - Sy^2 = 1$ with $R < S \leq 200$. For the system in the general case, Tzanakis [**11**] gave a method using elliptic curves and linear forms in elliptic logarithms. Ponnudurai [**9**] dealt with the Pell's equation $U^2 - 11V^2 = -2$ with the restrictions $Y^2 = 5 + 4U$ and $X^2 = 5 + 4V$. In [**6**], the system consisting of the Pell's equations $5y^2 - 20 = x^2$ and $2y^2 + 1 = z^2$ was considered. In [**7**], the concept of the characteristic number of two simultaneous Pell's equations was introduced by Mohanty and Ramasamy. A method for a set of Pell's

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