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A PEANO-AKÔ TYPE THEOREM FOR VARIATIONAL INEQUALITIES

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ABSTRACT. We consider in this paper a Peano-Akô property of solution sets in some quasilinear elliptic variational inequalities. As consequences, variants of that property and a partial Hukuhara-Kneser theorem for inequalities are derived.

1. Introduction. This paper is about a property of solution sets in variational inequalities. We consider here a variational version of the Peano-Akô property for solutions in inequalities. Roughly speaking, the property states that under certain conditions, the solutions of an equation “fill up” the “region” between certain specific solutions (maximal and minimal solutions in our case).

In classical versions of the Peano-Akô property, cf. e.g. [**1, 12**], this is expressed by the fact that the values of $u(x_0)$ of the solutions u at any point x_0 in the domain fill up the whole interval $[u_*(x_0), u^*(x_0)]$ where u_* and u^* are the minimal and the maximal solutions of the equation.

In the case of weak solutions, those functions may not be continuous and be only defined almost everywhere. This is particularly relevant for solutions of variational inequalities, as we know, cf. [**3, 11, 26**], that those functions are not continuous in general. Hence, the pointwise interpretation above is no longer valid for such solutions.

Peano-Akô type properties are related to the connectedness of solution sets (or parts of them), which is also known as a Hukuhara-Kneser type property, which states that the solution set (of a problem) is a continuum, i.e., a compact, connected set, in an appropriate function space. Hukuhara-Kneser type theorems have been derived in [**4, 18, 28–30**, see also the references therein], for ordinary differential equations, integral equations and parabolic equations and systems, the solutions of which are smooth in most cases. We are concerned here with the elliptic variational inequalities with solutions being non-necessarily smooth.

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