ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 36, Number 2, 2006

PERIODIC SOLUTIONS AND ASYMPTOTIC BEHAVIOR OF A PDE WITH HYSTERESIS IN THE SOURCE TERM

JANA KOPFOVÁ

ABSTRACT. A parabolic PDE with hysteresis in the source term is considered. The existence of periodic solutions for a general hysteresis operator is proven and an asymptotic result for solutions of this equation, using ideas due to Krejčí, is obtained.

1. Introduction. Let $\Omega \subset \mathbf{R}^N$, $N \geq 1$, be an open bounded set of Lipschitz class, denote by $\partial\Omega$ the boundary of Ω and set $Q := \Omega \times (0,\infty)$, $\Sigma := \partial\Omega \times (0,\infty)$.

In this paper we consider the following model equation

$$\frac{\partial u}{\partial t} - \triangle u + \mathcal{F}(u) = f \quad \text{in} \quad Q, \tag{1}$$

coupled with initial and boundary conditions, where

$$\mathcal{F} : M\left(\Omega; C^0([0,\infty))\right) \longrightarrow M\left(\Omega; C^0([0,\infty))\right)$$

is a continuous operator with memory, $M\left(\Omega; C^0([0,\infty))\right)$ denotes the Fréchet space of (strongly) measurable functions $\Omega \to C^0([0,\infty))$ and f is a given function.

Sufficient conditions for the existence and uniqueness of solutions of (1) are well known and we present them in the next section.

We study the question of existence of periodic solutions of (1) as well as asymptotic behavior of solutions as $t \to \infty$. To our knowledge there are so far only two papers dealing with such problems, [**1**] and [**5**]. In [**1**] they investigated the asymptotic behavior, as $t \to \infty$, of both the solution of (1) and the corresponding memory term $\mathcal{F}(u)$, where $\mathcal{F}(u)$ is a hysteresis operator. They showed that under some

Received by the editors on January 1, 2000, and in revised form on August 11, 2003.
Research supported by Grant 201-02-P040 of the Czech Grant Agency.