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POWER SUBGROUPS OF SOME HECKE GROUPS

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ABSTRACT. Let $q > 3$ be an even integer and let $H(\lambda_q)$ be the Hecke group associated to q. Let m be a positive integer and $H^m(\lambda_q)$ the power subgroup of $H(\lambda_q)$. In this work the power subgroups $H^m(\lambda_q)$ are discussed. The Reidemeister-Schreier method and the permutation method are used to obtain the abstract group structure and generators of $H^m(\lambda_q)$; their signatures are then also determined.

1. Introduction. In [**4**], Erich Hecke introduced the groups $H(\lambda)$ generated by two linear fractional transformations

$$x(z) = -\frac{1}{z} \quad \text{and} \quad u(z) = z + \lambda,$$

where λ is a fixed positive real number. Let $y = xu$, i.e.,

$$y(z) = -\frac{1}{z+\lambda}.$$

E. Hecke showed that $H(\lambda)$ is Fuchsian if and only if $\lambda = \lambda_q = 2\cos(\pi/q)$, for $q = 3, 4, 5, \dots$, or $\lambda \geq 2$. We are going to be interested in the former case. These groups have come to be known as the *Hecke groups*, and we will denote them by $H(\lambda_q)$, for $q \geq 3$. Then the Hecke group $H(\lambda_q)$ is the discrete subgroup of PSL$(2, \mathbf{R})$ generated by x and y, and it is isomorphic to the free product of two finite cyclic groups of orders 2 and q. $H(\lambda_q)$ has a presentation

$$H(\lambda_q) = \langle x, y \mid x^2 = y^q = I \rangle \cong C_2 * C_q, \quad [\mathbf{1}]. \tag{1.1}$$

Also $H(\lambda_q)$ has the signature $(0; 2, q, \infty)$, that is, all the groups $H(\lambda_q)$ are triangle groups. The first several of these groups are $H(\lambda_3) =$

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