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MOUFANG LOOPS THAT SHARE ASSOCIATOR AND THREE QUARTERS OF THEIR MULTIPLICATION TABLES

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ABSTRACT. Two constructions due to Drápal produce a group by modifying exactly one quarter of the Cayley table of another group. We present these constructions in a compact way, and generalize them to Moufang loops, using loop extensions. Both constructions preserve associators, the associator subloop and the nucleus. We conjecture that two Moufang 2-loops of finite order n with equivalent associator can be connected by a series of constructions similar to ours and offer empirical evidence that this is so for $n = 16$, 24, 32, the only interesting cases with $n \leq 32$. We further investigate the way the constructions affect code loops and loops of type $M(G, 2)$. The paper closes with several conjectures and research questions concerning the distance of Moufang loops, classification of small Moufang loops, and generalizations of the two constructions.

1. Introduction. Moufang loops, i.e., loops satisfying the *Moufang identity* $((xy)x)z = x(y(xz))$, are surely the most extensively studied loops. Despite this fact, the classification of Moufang loops is finished only for orders less than 64, and several ingenious constructions are needed to obtain all these loops. The purpose of this paper is to initiate a new approach to finite Moufang 2-loops. Namely, we intend to decide whether all Moufang 2-loops of given order with equivalent associator can be obtained from just one of them, using only group-theoretical constructions. (See below for details). We prove that this is the case for $n = 16$, 24, and 32, which are the only orders $n \leq 32$ for which there are at least two non-isomorphic nonassociative Moufang loops (5, 5, and 71, respectively). We also show that for every $m \geq 6$ there exist classes of loops of order 2^m that satisfy our hypothesis. Each of these classes consists of code loops whose nucleus has exactly two elements, cf. Theorem 8.8.

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