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ASYMPTOTIC BEHAVIOR OF PERIODIC COMPETITION DIFFUSION SYSTEM

YANBIN TANG AND LI ZHOU

ABSTRACT. In this paper, we consider the existence and attraction of positive periodic solution of a competition diffusion system. We first construct a pair of upper and lower solutions, then use the periodic comparison existence theorem to get a pair of T-periodic solutions $(\overline{u}, \underline{v})$ and $(\underline{u}, \overline{v})$. Finally we give a sufficient condition of $(\overline{u}, \underline{v}) = (\underline{u}, \overline{v})$ to answer the open question described by Ahmad and Lazer.

1. Introduction. The periodic competition diffusion system with no-flux boundary conditions

(1.1)

$$\begin{aligned} u_t &= \Delta u + u[a(x,t) - b(x,t)u - c(x,t)v], \\ v_t &= \Delta v + v[d(x,t) - e(x,t)u - f(x,t)v], \end{aligned} \qquad (x,t) \in \Omega \times [0, +\infty),$$

$$\frac{\partial u}{\partial n} = \frac{\partial v}{\partial n} = 0, \qquad (x,t) \in \partial\Omega \times [0, +\infty),$$

models the two species competition diffusion phenomena in a periodic changing environment, the coefficients $a(x,t), b(x,t), \ldots, f(x,t)$ are sufficiently smooth functions defined on a cylinder $\Omega \times [0, +\infty)$, where Ω is a smooth bounded domain in R^n. We assume that $a(x,t), \ldots, f(x,t)$ are strictly positive and periodic in the time variable t with period $T > 0$, and set

$$a_L = \min_{\overline{\Omega} \times [0,T]} a(x,t), \qquad a_M = \max_{\overline{\Omega} \times [0,T]} a(x,t), \ldots,$$

$$f_L = \min_{\overline{\Omega} \times [0,T]} f(x,t), \qquad f_M = \max_{\overline{\Omega} \times [0,T]} f(x,t).$$

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