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SYMBOLIC POWERS OF RADICAL IDEALS

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ABSTRACT. Hochster proved several criteria for the case when for a prime ideal P in a commutative Noetherian ring with identity, $P^n = P^{(n)}$ for all n. We generalize the criteria to radical ideals.

1. Introduction. In [**1**], Hochster established several criteria for the case when for a prime ideal P in a Noetherian ring R, the nth power P^n of P equals the nth symbolic power $P^{(n)}$ of P for every positive integer n. He used a so-called test sequence of ideals in a polynomial ring over R to determine whether $P^n = P^{(n)}$ for all n. We extend Hochster's criteria to radical ideals.

Here is the set-up: let R be a Noetherian domain and P an ideal of R. Suppose that $\{a_1, a_2, \dots, a_m\}$ is a generating set for P. Write the m-tuple as $\underline{\mathbf{p}} = (a_1, a_2, \dots, a_m)$. Let $S = R[x_1, x_2, \dots, x_m]$, where $x_1, x_2, \dots, x_m$ are indeterminates over R.

Definition 1.1. For an ideal $P = (a_1, \dots, a_m)R$ of R, define recursively ideals of $S = R[x_1, \dots, x_m]$:

$$J_0(\underline{\mathbf{p}}) = 0$$

and

$$J_{n+1}(\underline{\mathbf{p}}) = \left(\left\{\Sigma_{i=1}^m s_i x_i \;\middle|\; s_i \in S \text{ and } \Sigma_{i=1}^m s_i a_i \in J_n(\underline{\mathbf{p}})\right\}\right) S$$

for $n \geq 0$. We write J_n for $J_n(\underline{\mathbf{p}})$ and denote $J = \cup_{n=1}^\infty J_n$. We call the sequence of ideals

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