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A STABILITY THEOREM FOR A CLASS OF DISTRIBUTED PARAMETER CONTROL SYSTEMS

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ABSTRACT. This paper presents an optimal control problem governed by a hyperbolic equation. The control may appear in the cost functional and in the right side of this equation. The difference approximations problem for the considered problem is obtained. A stability estimate of the difference approximations problem is established.

1. Introduction. Very recently the optimal control distributed parameter systems has received the attention of many control engineers. Many of the problems of control in air-frames design, shipbuilding, nuclear reactors, magnetohydrodynamics and other engineering fields [**4, 9**] are problems of control of systems with distributed parameters, and, therefore, are more difficult to optimize. The first serious work in this direction was introduced by Botkovsky and Lerner [**2, 3**] and Butkovsky [**1**]. Warng [**16**] and Rehbock [**10**] attempted to present a general discussion of various problems associated with the control of distributed parameter systems. Chaudhuri [**5, 11**] discussed the derivation of a maximum principle and obtained the optimal control function through the discretization schemes and via the method of gradients and quasilinearization techniques for a class of hyperbolic partial differential equations. Farag [**6**] discussed the existence and uniqueness theorem, the sufficient differentiability conditions of the cost functional and its gradient formulae based on solving the adjoint system and the necessary optimality conditions for a class of hyperbolic partial differential equations.

This paper presents an optimal control problem governed by a hyperbolic equation. The control may be act in the cost functional and in the right side of this equation. The difference approximations prob-

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