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ASYMPTOTIC APPROXIMATIONS OF EIGENVALUES AND EIGENFUNCTIONS FOR REGULAR STURM-LIOUVILLE PROBLEMS

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ABSTRACT. In this paper, we obtain estimates for the eigenvalues and eigenfunctions associated with Sturm-Liouville equation $y'' + (\lambda - q)y = 0$ having regular endpoints under the condition that $q \in L^1[0, \pi]$.

1. Introduction. We consider the differential equation

$$y''(t) + (\lambda - q(t))y(t) = 0, \tag{1}$$

defined on the interval $[0, \pi]$ where $q \in L^1[0, \pi]$. We impose the following boundary conditions

$$y(0)\sin\alpha + y'(0)\cos\alpha = 0 \tag{2}$$

$$y(\pi)\sin\beta + y'(\pi)\cos\beta = 0 \tag{3}$$

where α and β are real numbers. In the case $\beta = \pi/2$, the boundary condition at π is known as a Dirichlet boundary condition, and the case $\beta = 0$ known as a Neumann boundary condition. We make the point that a more general second-order differential equation

$$\{p(t)y'(t)\}' + \{\lambda s(t) - q(t)\}y(t) = 0 \tag{4}$$

can be reduced to an equation of type (1) by using the Liouville transformation if p'' and s'' exist and are piecewise continuous [**3**]. In this case, the boundary conditions (2)–(3) do not change their form.

The derivation of asymptotic approximations of eigenvalues for regular Sturm-Liouville problems has a long history. Motivation for studying eigenvalues and eigenfunctions arise from different types of problems

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