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LOCAL CONNECTEDNESS IN HYPERSPACES

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ABSTRACT. Variants of local connectedness as local connectedness, local arcwise connectedness, strong local connectedness and strong local arcwise connectedness at a point are studied for the following hyperspaces of a compact Hausdorff space X: $C_n(X)$, $C_\infty(X)$, $F_n(X)$, $F_\infty(X)$ and 2^X.

1. Introduction. In [**20**] it is proved that if X is a Hausdorff compact space, then local connectedness and local arcwise connectedness of the hyperspace $C(X)$ of all subcontinua of X and of the hyperspace 2^X of all nonempty closed subsets of X are equivalent at any point. In [**21**] it is shown that for metric spaces the above properties are equivalent to another one, namely to local k-connectedness. In the present paper the above equivalences are studied for further hyperspaces: $C_n(X)$ of all members of 2^X that have no more than n components, $C_\infty(X)$ of all members of 2^X that have finitely many components, $F_n(X)$ of all members of 2^X that have no more than n points, and $F_\infty(X)$ of all members of 2^X that consist of finitely many points. The obtained results complete not only the above mentioned papers [**20**] and [**21**], but also a number of other ones related to the same topic of local connectivity properties of hyperspaces as, e.g., [**4–7, 9–12, 18, 19**] and others.

The paper consists of six sections. In the first one we collect, for reader information and completeness of this paper, some known results about local connectedness at a point of the hyperspace 2^X, i.e., at a nonempty closed subset of X. The second and the third sections are devoted to variants of local connectedness at a point of the hyperspace $C_n(X)$. We study local connectedness, local arcwise connectedness, strong local connectedness and strong local arcwise connectedness of

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