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DIFFERENTIAL INCLUSIONS ON PROXIMATE RETRACTS OF SEPARABLE HILBERT SPACES

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ABSTRACT. New existence results are presented which guarantee the existence of viable solutions to differential inclusions in separable Hilbert spaces. Our results rely on the existence of maximal solutions for an appropriate differential equation in the real case.

1. Introduction. In this paper we discuss the existence of solutions $y : [0, T] \to K \subseteq H$ (so called viable solutions) to the differential inclusion

$$(1.1) \qquad \begin{cases} y'(t) \in \phi(t, y(t)) & \text{a.e.} \quad t \in [0, T] \\ y(0) = x_0 \in K. \end{cases}$$

Here $T > 0$ is fixed, K is a proximate retract (defined in Section 2) and H is a separable Hilbert space. Our existence theory relies on (i) solution set results for differential inclusions due to Cichon and Kubiaczyk [**2**], (ii) the existence of maximal solutions for appropriate differential equations in the real case, (iii) properties of the Bouligand cone and (iv) the Urysohn function. Our results extend and complement results in the literature (see [**3–5, 7, 8**] and the references therein).

For the convenience of the reader we recall the results in [**2**]. Consider the differential inclusion

$$(1.2) \qquad \begin{cases} y'(t) \in F(t, y(t)) & \text{a.e.} \quad t \in [0, T] \\ y(0) = x_0 \in H \end{cases}$$

where $F : [0, T] \times H \to C(H)$ (here $C(H)$ denotes the family of nonempty compact subsets of H) and H is a separable Hilbert space. We look for solutions to (1.2) in $W^{1,1}([0, T], H)$. Recall $W^{1,1}([0, T], H)$ denotes the Sobolev class of absolutely continuous functions on $[0, T]$. We assume F satisfies *some* of the following conditions, to be specified

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