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REAL RANK OF C^*-TENSOR PRODUCTS WITH THE C^*-ALGEBRA OF BOUNDED OPERATORS

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ABSTRACT. We show under an assumption on the real rank zero that the real rank of the minimal C^*-tensor products of unital exact C^*-algebras with the C^*-algebra of bounded operators is less than or equal to one. Moreover, several consequences of this result are obtained.

1. Introduction. The real rank for C^*-algebras was introduced by Brown and Pedersen [**3**]. This notion has been quite important in the theory of C^*-algebras such as the classification theory of C^*-algebras, cf. [**9**] and its reference. On the other hand, some basic formulas for the real rank has been obtained by [**1, 3, 6, 10, 11, 15**], etc. However, it is hard to compute the real rank of C^*-algebras in some general situations so that some desirable formulas for the real rank has not been proven yet. For example, the real rank formula for C^*-tensor products has not been obtained completely.

In this paper we obtain a real rank formula for the minimal C^*-tensor products of unital exact C^*-algebras with the C^*-algebra of bounded operators under an assumption on the real rank zero. The main idea of the proof is a modification (to the real rank case) of Rieffel's proof for the stable rank formula [**16**, Theorem 6.4] for C^*-tensor products by the C^*-algebra of compact operators. However, the process of the real rank case is more complicated than the stable rank case as shown in Theorem 1. As a consequence, several results of the real rank of C^*-tensor products are obtained by using the results of Kodaka-Osaka [**10, 11, 15**], Zhang [**24**] and Lin [**13**]. Also, the real rank formula in Theorem 1 would be useful in other situations in the future. See [**5, 17–22**] for some related works.

Notation. Let $\mathbf{B}(H)$ be the C^*-algebra of all bounded operators on a separable infinite-dimensional Hilbert space H, and let $\mathbf{K}$ be the C^*-

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