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# TOPOLOGICAL TRIVIALITY OF FAMILY OF FUNCTIONS AND SETS

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ABSTRACT. In this article we divulge one of the subjects most interesting in the Theory of Singularities, to know, the topological triviality of families of functions and sets. We present, briefly, definitions and some of the results most important when treating of real singularities.

**1. Introduction.** Given an object $V$ in the Euclidean space $\mathbf{R}^3 = \mathbf{R}^2 \times \mathbf{R}$, we can look at $V$ as a 1-parameter family of objects in $\mathbf{R}^2$. In fact, let us consider $V_t = \{(x,y) : (x,y) \in V\}$. This is a classic perspective which appears, for example, in the principle of Cavallieri used for the calculation of volumes.

A technique very common in singularity theory explores the inverse way described above. For example, Let $f\colon \mathbf{R}^3 = \mathbf{R}^2 \times \mathbf{R} \to \mathbf{R}$; $f(0 \times \mathbf{R}) = 0$ be a family of functions $f_t\colon \mathbf{R}^2 \to \mathbf{R}$; $f_t(x,y) = f(x,y,t)$. Associated to this family we have the following family of sets $X_t = f^{-1}(0)$ in $\mathbf{R}^2$. Then, from the object $X = f^{-1}(0) \in \mathbf{R}^3$, it is possible to know the local topological property of the family $X_t$. In this context, Whitney introduced the following concept of regularity of an analytic family of analytic sets $X_t$ in $\mathbf{R}^n$, through $0 \in \mathbf{R}$, with the property that the singular set of $X = \{(x,t) : x \in X_t\}$ is contained in $Y = 0 \times \mathbf{R}$ (t-axis) and $X - Y$ is a smooth analytic subset in $\mathbf{R}^n \times \mathbf{R}$ which is dense in $X$.

(a) *Condition. $X$ is $(a)$-regular on the $t$-axis $Y$.* If, for each $y \in Y$, the following holds: if $(p_i)$ is a sequence in $X - Y$; $p_i \to y$, and the sequence of tangent planes $T_{p_i}(X - Y)$ converges to a plane $\tau$ (in the appropriated Grassmaniann), then $T_y Y \subset \tau$.

(b) *Condition. $X$ is $(b)$-regular on the $t$-axis $Y$.* If, for each $y \in Y$, the following holds: if $(p_i)$ and $(y_i)$ are pairs of sequence of points in $X - Y$ and $Y$ respectively; $p_i \to y$, $y_i \to y$ and the sequence of lines $p_i y_i$

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