

NEW CONVERGENCE THEOREMS FOR CERTAIN ITERATIVE SCHEMES IN BANACH SPACES

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ABSTRACT. In this paper, we first review some convergence results for certain iterative schemes for a certain class of operators and discuss essential relations between the old results and new results. Then we establish several general convergence principles for certain iterative schemes for accretive operators, and show how our convergence principles can be applied to Mann's and Ishikawa's methods. Finally, four open problems are also given.

1. Introduction. The main purpose of this paper is to present a version for the convergence theorems due to Nevanlinna and Reich [35], Bruck and Reich [6]. As consequences of the revision, most of the recent results can be deduced from our convergence principle.

Early in 1979, Nevanlinna and Reich [35] studied discrete implicit and explicit schemes for finding the zeros of accretive operators that satisfy the convergence condition ([35]). Soon afterwards, Bruck and Reich [6] generalized the results of Nevanlinna and Reich [35] by establishing the general convergence principle for the implicit and explicit schemes for a certain class of operators that satisfy the so called condition 2.1 in [6].

Let $A : D(A) \subset E \rightarrow 2^E$ be an m -accretive operator in a Banach space E . The following implicit scheme with errors and the explicit scheme were considered by Nevanlinna and Reich [35]:

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