

GENERALIZED FREE PRODUCTS OF RESIDUALLY p -FINITE GROUPS

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ABSTRACT. In this note, we characterize the residual p -finiteness of generalized free products and tree products of certain residually p -finite groups with non-trivial center amalgamating infinite cyclic subgroups and the tree products of certain one-relator groups. We then apply our results to tree products of finitely generated torsion-free nilpotent groups and free groups.

1. Introduction. Let p be a prime. A group G is said to be residually p -finite if for each non-trivial element x of G , there exists a normal subgroup N of index a power of p in G such that $x \notin N$. It is well known that free groups and finitely generated torsion-free nilpotent groups are residually p -finite for all primes p (Iwasawa [5], Gruenberg [3]). In [4], Higman proved that a generalized free product of two finite p -groups amalgamating a cyclic subgroup, is residually p -finite. Kim and McCarron [6] then generalized Higman's result by proving that the generalized free product of residually p -groups amalgamating a finite cyclic subgroup, is residually p -finite. In the same paper [6], they also proved a sufficient condition for a free product of finitely many residually p -finite groups amalgamating a single infinite cyclic subgroup, to be residually p -finite. From this, they showed that a generalized free product of finitely many free groups or finitely generated torsion-free nilpotent groups amalgamating a maximal cyclic subgroup is residually p -finite for all primes p . In [11], Wong and Tang extended Kim and McCarron's result to finite tree product of residually p -finite groups, amalgamating infinite cyclic subgroups. Thus, the finite tree products of finitely many free groups or finitely generated torsion-free nilpotent groups amalgamating maximal cyclic subgroups are residually p -finite for all primes p .

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