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A MASSERA THEOREM FOR QUASI-LINEAR PARTIAL DIFFERENTIAL EQUATIONS OF FIRST ORDER

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ABSTRACT. Massera-type criteria are derived for the existence of periodic wave solutions of quasi-linear partial differential equations of first order. Results generalize a theorem of Massera for first order scalar ordinary differential equations.

1. Introduction. In 1950, Massera [**8**] first established the following results, which now are often referred to as Massera theorems.

Theorem A. *For a scalar differential equation*

$$\dot{x} = f(t, x), \tag{1}$$

where $f \in C(\mathbf{R}_+ \times \mathbf{R} \to \mathbf{R})$ *is* ω*-periodic in* t *for some* $\omega > 0$*, the existence of a solution that is bounded in the future implies the existence of a nonconstant* ω*-periodic solution.*

Theorem B. *Consider a linear system of differential equation*

$$\dot{x} = A(t)x + b(t), \tag{2}$$

where $A \in C(\mathbf{R} \to \mathbf{R}^{n\times n})$ *and* $b \in C(\mathbf{R} \to \mathbf{R}^n)$ *are* ω*-periodic for some* $\omega > 0$*. System* (2) *admits a nonconstant* ω*-periodic solution if and only if it admits a solution that is bounded in the future.*

In 1973, Chow [**1**] extended Theorem B to linear scalar functional differential equations with finite delay of retarded type under a "small

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