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PHELPS' UNIQUENESS PROPERTY FOR $K(X,Y)$ IN $L(X,Y)$

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ABSTRACT. We study pairs of Banach spaces X and Y with X^* or Y^* having a metric compact approximation of the identity (MCAI) with adjoint operators such that the subspace $K(X,Y)$ of compact operators from X to Y has the Phelps' uniqueness property U in the space of all continuous linear operators $L(X,Y)$, i.e., every functional $f \in K(X,Y)^*$ has a unique norm-preserving extension to $L(X,Y)$.

Our main results are: (1) $K(X,X)$ has property U in $L(X,X)$ whenever X has an MCAI and $K(E,E)$ has property U in $L(E,E)$ for every closed separable subspace E of X having an MCAI; (2) if a Banach space Y has an MCAI, then $K(X,Y)$ has property U in $L(X,Y)$ for all Banach spaces X if and only if $K(l_1,Y)$ has property U in $L(l_1,Y)$. We also show that if a separable dual space X^* has an MCAI with adjoint operators, then property U for $K(X,X)$ in $L(X,X)$ is determined by the properties of the extreme points of the unit ball of $L(X,X)^*$.

0. Introduction. Let X be a (real or complex) Banach space, and let Z be a closed subspace of X. By the Hahn-Banach theorem, every continuous linear functional $g \in Z^*$ has a norm-preserving extension $f \in X^*$. In general, such an extension is highly non-unique. Following Phelps [**16**], we say that Z has *property* U in X if every $g \in Z^*$ has a unique norm-preserving extension $f \in X^*$.

According to the terminology in [**2**], a closed subspace Z of a Banach space X is said to be an *ideal* in X if there exists a contractive projection P on X^* with $\ker P = Z^\perp$. It is straightforward to verify that, if Z is an ideal in X, then, for every $f \in X^*$, $Pf \in X^*$ is a norm-preserving extension of the restriction $f|_Z \in Z^*$. Therefore, $\operatorname{ran} P$ is canonically isometric to Z^*. In the sequel, we shall use the (generally non-Hausdorff) weak topology $\sigma(X, \operatorname{ran} P)$. Ideals with property U have been studied e.g. in [**10, 11, 14, 15**].

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