**BOCKY MOUNTAIN** JOURNAL OF MATHEMATICS Volume 36, Number 5, 2006

## SHARP ESTIMATES FOR SOME ITERATED OPERATORS IN ORLICZ SPACES

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ABSTRACT. In [7] and [6] sharp Orlicz estimates for the maximal and conjugate functions on the one-dimensional torus were given. Starting from their results we describe the pairs of growth functions  $(\psi,\varphi)$  such that modular  $L^\psi \to L^\phi$ estimates hold for both, the strong maximal function and the n<sup>th</sup>-iteration of the Hilbert transform. We also show that our conditions are sharp. These results are achieved in a setting general enough to include both operators.

**1.** Introduction. The behavior of classical operators in Orlicz spaces has been extensively studied by many authors, see, for instance, [1-4, 8]. Whenever we have an operator bounded on  $L^p$  for p ranging on some interval, usually it is not bounded on some of the end points in the sense of the *p*-norm, although it satisfies some weaker estimates. In such situation the behavior of the operator near those extreme values is better understood when we refine the  $L^{p}$ -family of spaces by introducing the richer class of Orlicz-spaces.

More precisely, the kind of questions to answer here are the following: given an operator T and some Orlicz space, say  $L^{\psi}$ , which is the optimal local integrability for all the functions in  $T(L^{\psi})$ ? Or further, when the underlying measure space has finite measure, which is the optimal space  $L^{\phi}$  such that T is bounded from  $L^{\psi}$  into  $L^{\phi}$ ?

Results in this direction may be found in [7] for the Hardy-Littlewood maximal function on the torus, in [6] for fractional maximal and integral operators in any dimension and the conjugate function, and in [4] for generalized Hardy operators.

In this paper we shall be concerned with the "iterated" Hilbert transform and the strong maximal function on the n-dimensional torus

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<sup>2000</sup> AMS Mathematics Subject Classification. Primary 42B25.

Key words and phrases. Iterated operators, Orlicz spaces. The authors were supported by the Consejo Nacional de Investigaciones Científicas y Técnicas de la República Argentina and by the Universidad Nacional del Litoral.

Received by the editors on Jan. 21, 2003, and in revised form on Nov. 4, 2003.