# SHARP ESTIMATES FOR SOME
# ITERATED OPERATORS IN ORLICZ SPACES

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ABSTRACT. In [**7**] and [**6**] sharp Orlicz estimates for the maximal and conjugate functions on the one-dimensional torus were given. Starting from their results we describe the pairs of growth functions $(\psi, \varphi)$ such that modular $L^\psi \to L^\phi$ estimates hold for both, the strong maximal function and the $n^{th}$-iteration of the Hilbert transform. We also show that our conditions are sharp. These results are achieved in a setting general enough to include both operators.

**1. Introduction.** The behavior of classical operators in Orlicz spaces has been extensively studied by many authors, see, for instance, [**1–4, 8**]. Whenever we have an operator bounded on $L^p$ for $p$ ranging on some interval, usually it is not bounded on some of the end points in the sense of the $p$-norm, although it satisfies some weaker estimates. In such situation the behavior of the operator near those extreme values is better understood when we refine the $L^p$-family of spaces by introducing the richer class of Orlicz-spaces.

More precisely, the kind of questions to answer here are the following: given an operator $T$ and some Orlicz space, say $L^\psi$, which is the optimal local integrability for all the functions in $T(L^\psi)$? Or further, when the underlying measure space has finite measure, which is the optimal space $L^\phi$ such that $T$ is bounded from $L^\psi$ into $L^\phi$?

Results in this direction may be found in [**7**] for the Hardy-Littlewood maximal function on the torus, in [**6**] for fractional maximal and integral operators in any dimension and the conjugate function, and in [**4**] for generalized Hardy operators.

In this paper we shall be concerned with the "iterated" Hilbert transform and the strong maximal function on the $n$-dimensional torus