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FULL ELASTICITY IN ATOMIC MONOIDS AND INTEGRAL DOMAINS

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ABSTRACT. Let M be a commutative cancellative atomic monoid and M^* its set of nonunits. Let $\rho(x)$ denote the elasticity of factorization of $x \in M^*$, $\mathcal{R}(M) = \{\rho(x) \mid x \in M^*\}$ the set of elasticities of elements of M, and $\rho(M) = \sup \mathcal{R}(M)$ the elasticity of M. We say M is *fully elastic* if $\mathcal{R}(M) = \mathbf{Q} \cap [1, \rho(M)]$. We call an atomic integral domain D fully elastic if its multiplicative monoid, denoted $D^\bullet$, is fully elastic. We examine the full elasticity property in the context of Krull monoids with finite divisor class groups, numerical monoids and certain integral domains. For every real number $\alpha \geq 1$, we construct a fully elastic Dedekind domain D with $\rho(D) = \alpha$. In particular, while we show that noncyclic numerical monoids are never fully elastic, we do verify that several large classes of Krull monoids, and hence certain Krull domains, are fully elastic.

1. Introduction and definitions. Let M be a commutative cancellative monoid with M^* its set of nonunits and $\mathcal{A}(M)$ its set of irreducibles (or atoms). We suppose M is *atomic* (i.e., every element of M^* is a sum of atoms). Much recent literature has been devoted to the study of monoids in which elements fail to factor uniquely. In particular, a central topic of this work has been the *elasticity* of elements of M, which measures their failure to factor uniquely. While much is known about the supremum of the set of elasticities, we study here the complete set of elasticities in several important classes of monoids and integral domains.

We begin with some definitions and notations. For $x \in M^*$, define

$$\mathcal{L}(x) = \{n \mid x = \alpha_1, \dots, \alpha_n \text{ with each } \alpha_i \in \mathcal{A}(M)\}$$

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