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ON CERTAIN STRUCTURES DEFINED ON THE TANGENT BUNDLE

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ABSTRACT. The differential geometry of tangent bundles was studied by several authors, for example: Davies [**4**], Yano and Davies [**5**], Dombrowski [**6**], Ledger and Yano [**9**] and Blair [**1**], among others. It is well known that an almost complex structure defined on a differentiable manifold M of class C^∞ can be lifted to the same type of structure on its tangent bundle $T(M)$. However, when we consider an almost contact structure, we do not get the same type of structure on $T(M)$. In this case we consider an odd dimensional base manifold while our tangent bundle remains to be even dimensional. The purpose of this paper is to examine certain structures on the base manifold M in relation to that of the tangent bundle $T(M)$.

1. Introduction. Let M be an n-dimensional differentiable manifold, and let $T(M)$ be its tangent bundle. Then $T(M)$ is also a differentiable manifold of dimension $2n$ [**11**]. Let $X = \sum_{i=1}^n x^i(\partial/\partial x^i)$ and $\omega = \sum_{i=1}^n \omega^i\, dx^i$ be the expressions in local coordinates for the vector field X and the 1-form ω in M. Let $(x^i,\, y^i)$ be local coordinates of a point in $T(M)$ induced naturally from the coordinate chart $(U,\, x^i)$ in M. If f is a function in M, then its vertical lift f^V is a function in $T(M)$ obtained by forming the composition of $\pi : T(M) \to M$ and $f : M \to R$ so that

$$f^V = f \circ \pi. \tag{1.1}$$

The complete lift f^C of f is also a function in $T(M)$ given by

$$f^C = y^i\, \partial_i f = \partial f. \tag{1.2}$$

We can introduce a system of local coordinates $\left(x^h, y^h\right)$ in an open set $\pi^{-1}(U) \subset T(M)$. Here we call $\left(x^h, y^h\right)$ the coordinates in $\pi^{-1}(U)$

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