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ON A CLASS OF TWO-POINT BOUNDARY VALUE PROBLEMS WITH SINGULAR BOUNDARY CONDITIONS

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ABSTRACT. A new existence theory for a class of second order two-point boundary value problems with nonlinear boundary conditions which can blow up in finite intervals is established. The proofs are based on the dual variational principle and the critical point theory.

1. Introduction. In this paper we consider second order mixed boundary value problems of the type

$$\begin{cases} (1/p)(py')' + f(t,y) = 0 & 0 < t < 1, \\ \lim_{t\to 0^+} p(t)y'(t) = 0, \\ \lim_{t\to 1^-} p(t)y'(t) + g(y(1)) = 0, \end{cases} \tag{1.1}$$

where $p > 0$ and g is defined only on a bounded open interval, with singularities at the extremities. A typical situation is when $g : (a,b) \to \mathbf{R}$, is increasing, where

$$-\infty < a < 0 < b < \infty$$

and

$$\lim_{y\to a^+} g(y) = -\infty \quad \text{and} \quad \lim_{y\to b^-} g(y) = +\infty. \tag{1.2}$$

In recent years, the problem (1.1) when $f(t,y)$ is singular at $y = 0$ and the boundary conditions are linear has been studied extensively. We refer the reader to the books [**2, 14**] for up-to-date information on the subject. Current research on this type of problem has been mainly

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