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BELLMAN FUNCTIONS AND MRA WAVELETS

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ABSTRACT. In this paper, we discuss how far the method of Bellman functions can be generalized from use with Haar functions to use with general MRA wavelets.

1. Introduction. The method of Bellman functions is a very powerful method in harmonic analysis which has been used to prove a lot of interesting results, see [**3**–**7**] for just a few. In its original form, it is intimately connected with the Haar function system, as it is used to estimate sums involving Haar coefficients. A natural question arises: Can the method be generalized to work with coefficients of general multi-resolution-analysis wavelets? This paper strives to answer this question. The answer is yes, but the Bellman function will be much more difficult to find in this general case.

The method itself, namely the proof of the result given the Bellman function, generalizes with only one change: The inequality conditions we need the Bellman function to satisfy are summed versions of the conditions in the Haar wavelet case. The big difference comes in the application of the method, namely when we are looking for the Bellman function. In the Haar case, the scaling sequence has only positive terms. In general, the terms can be negative. This means that we cannot use the Cauchy-Schwarz inequality to define the domain of the Bellman function, and it also makes the usual differential inequality difficult to work with for general wavelets.

In this paper, we will first present a simple Bellman proof of the bound of the discrete square function, which is based on the Haar function system. We then discuss which aspects of a Haar-based proof need to be adjusted when we are working with wavelets. To illustrate the Bellman method when used with wavelets, we then give a Bellman proof of the bound of the wavelet square function.

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