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THE SQUARE OF A MAP, SYMBOLIC DYNAMICS AND THE CONLEY INDEX

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ABSTRACT. We consider a map f from a locally compact metric space to itself, and use the discrete Conley index to study the difference between the local dynamics of f and f^2. In particular, we present a method, based on work by Mischaikow, Szymczak, et al., for detecting positive entropy symbolic dynamics by measuring the difference between Conley indices for f and f^2.

1. Introduction. Let $f : X \to X$ be a continuous map of a locally compact metric space and N a compact subset of X. Any point that stays in N under all forward and backward iterates of f certainly does so for f^2 as well, but the converse is not true; thus, the maximal invariant set in N under f^2 contains the corresponding set under f, see Section 2 for exact definitions. In this paper we use the discrete Conley index to study the extent to which the two sets differ.

In particular, we present a method, based on work by Mischaikow, Szymczak, et al. [**2, 16**], for detecting symbolic dynamics by measuring the difference between Conley indices for f and f^2. We see that the nonnilpotence of certain products of the induced maps on homology corresponds to the existence of positive entropy renewal systems. A consequence is that if an invariant set satisfies certain decomposability assumptions and a homology map on the Conley index for f has a nonzero eigenvalue whose square is not an eigenvalue for the corresponding map for f^2, then f has positive topological entropy.

Sections 2 and 3 contain background information, Section 2 on the Conley index and Section 3 on renewal systems. In Section 4 we discuss some basic results on the differences between the local dynamics for f and f^2. Finally, in Section 5 we discuss the method for detecting symbolic dynamics.

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