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CLASSICAL OPERATORS ON MIXED-NORMED SPACES WITH PRODUCT WEIGHTS

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ABSTRACT. We prove norm inequalities for a variant of the Hardy-Littlewood maximal function on weighted mixed-norm spaces. These results are applied to singular integral operators, including the double Hilbert transform.

1. Introduction. Let f be a locally integrable function on $\mathbf{R}^n$. We define the *Hardy-Littlewood maximal function* Mf of f by

$$Mf(x) = \sup \frac{1}{|Q|} \int_Q |f(y)|\, dy,$$

where the supremum is taken over all cubes $Q \subset \mathbf{R}^n$ containing x. In 1930, Hardy and Littlewood proved that this operator is bounded on L^p for $1 < p \le \infty$. This result has been generalized in many directions. Fefferman and Stein [**4**] proved a vector-valued version:

(1.1)
$$\left(\int_{\mathbf{R}^n} \left(\sum_{j=1}^{\infty} |Mf_j(x)|^q \right)^{p/q} dx \right)^{1/p} \le C \left(\int_{\mathbf{R}^n} \left(\sum_{j=1}^{\infty} |f_j(x)|^q \right)^{p/q} dx \right)^{1/p}$$

for $1 < p,\ q < \infty$. A key element of their proof is a weighted-norm inequality:

$$\left(\int_{\mathbf{R}^n} |Mf(x)|^p w(x)\, dx \right)^{1/p} \le C \left(\int_{\mathbf{R}^n} |f(x)|^p Mw(x)\, dx \right)^{1/p}$$

which holds for any $p > 1$. If there is a constant $C > 0$ so that $Mw(x) \le Cw(x)$, which is known as the A_1 condition, then we have

$$\left(\int_{\mathbf{R}^n} |Mf(x)|^p w\, dx \right)^{1/p} \le C \left(\int_{\mathbf{R}^n} |f(x)|^p w\, dx \right)^{1/p} = \|f\|_{p,w}.$$

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