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ON IMPULSIVE TIME-VARYING SYSTEMS WITH UNBOUNDED TIME-VARYING POINT DELAYS: STABILITY AND COMPACTNESS OF THE RELEVANT OPERATORS MAPPING THE INPUT SPACE INTO THE STATE AND OUTPUT SPACES

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ABSTRACT. This paper is concerned with time-varying systems with non-necessarily bounded everywhere continuous time-differentiable time-varying point delays. The delay-free and delayed dynamics are assumed to be time-varying and impulsive, in general, and the external input may be impulsive as well. For given initial conditions, the (unique) homogeneous state-trajectory and output trajectory are equivalently constructed from three different auxiliary homogeneous systems, the first one being delay-free and time-invariant, the second one possessing the delay-free dynamics of the current delayed system and the third one being the homogeneous part of the system under study. In this way, the constructed solution trajectories of both the unforced and forced systems are obtained from different (input-state space/output space and state space to output space) operators. The stability of the homogeneous auxiliary system and that of the object system are investigated. Finally, the compactness of some of the various relevant operators involved in the descriptions of the solution trajectories is investigated.

1. Introduction. Time-delay systems have been widely investigated in the last years both in a theoretical context and in that of related applications, see for instance, [**2, 4, 6–12, 14–15, 17, 19–27**]. Those systems become inherently attractive from a theoretical point of view since they are described by (infinite-dimensional) functional equations and because of their interest towards potential applications like, for instance, population growth models, transportation, communications as well as war-peace and agricultural models [**6, 26**]. A wide variety of both dependent and independent (of delay) results exist, see for

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