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PERIODIC BOUNDARY VALUE PROBLEM FOR FIRST ORDER IMPULSIVE DIFFERENTIAL EQUATION AT RESONANCE

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ABSTRACT. We develop a general theorem concerning the existence of solutions to the periodic boundary value problem for the first-order impulsive differential equation,

$$\begin{cases} x'(t) = f(t, x(t)) & t \in J \setminus \{t_1, t_2, \dots, t_k\} \\ \triangle x(t_i) = I_i(x(t_i)) & i = 1, 2 \dots, k \\ x(0) = x(T). \end{cases}$$

And using it we get a concrete existence result. Moreover, to our knowledge, the coincidence degree method has not been used with first order impulsive differential systems. Besides, our results can also be applied in studying the usual periodic boundary value problem at resonance without impulses.

1. Introduction. In recent years, many authors have discussed impulsive differential equation, see [**1, 3, 6, 7, 9**]. For example, He and Ge [**6**], Bainov and Hristova [**1**] and Liz [**9**] investigated the existence of solutions for first order impulsive equations by use of upper and lower solution methods. Frigon and O'Regan [**3**] investigated the existence of solutions to first order impulsive equations by the alternative theorem and upper and lower solution method. Dong [**2**], Liu and Yu [**8**] researched the existence of solutions to second order impulsive equations by making use of the coincidence degree theory and autonomous curvature bound set. However, to our knowledge, the coincidence degree method developed by Gaines and Mawhin [**5**] has not been used to the first order impulsive differential systems. In this paper, we are concerned with the periodic boundary value problem for

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