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# ON A STATIONARY, TRIPLE-WISE INDEPENDENT, ABSOLUTELY REGULAR COUNTEREXAMPLE TO THE CENTRAL LIMIT THEOREM

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ABSTRACT. An earlier paper gave a construction of a strictly stationary, finite-state, nondegenerate random sequence which satisfies pairwise independence and absolute regularity but fails to satisfy a central limit theorem. Here it will be shown that random sequence is in fact triple-wise independent (though not quadruple-wise independent).

**1. Introduction.** Etemadi [**7**] proved the strong law of large numbers for sequences of pairwise independent, identically distributed random variables with finite absolute first moment. Janson [**9**] constructed several classes of (nondegenerate) strictly stationary sequences of pairwise independent random variables with finite second moments such that the CLT (central limit theorem) fails to hold. Subsequently, the author [**2**] constructed such an example with the additional property of absolute regularity (defined below). For the examples in those two papers, as well as other examples of pairwise independent sequences, Cuesta and Matrán [**4**] examined the behavior of the partial sums further, e.g., in connection with the law of the iterated logarithm. For an arbitrary integer $N \geq 3$, Pruss [**12**] constructed a (not strictly stationary) sequence of $N$-tuple-wise independent, identically distributed random variables with finite second moment such that the CLT fails to hold. In that paper, for a given $N \geq 3$, the existence of such examples that are also strictly stationary was left as an open question. For $N = 3$, the answer is affirmative. The purpose of this note is to show that the example in [**2**] alluded to above is triple-wise independent.

Suppose $X := (X_k,\, k \in \mathbf{Z})$ is a strictly stationary sequence of random variables on a probability space $(\Omega, \mathcal{F}, P)$. For $-\infty \leq j \leq l \leq \infty$, let $\mathcal{F}_j^l$ denote the $\sigma$-field ($\subset \mathcal{F}$) of events generated by the random variables

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