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ON LOCALLY UNIFORMLY A-PSEUDOCONVEX ALGEBRAS

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ABSTRACT. Conditions when a unital locally uniformly A-pseudoconvex algebra (E, τ) is (or when there exists a topology τ' on E such that (E, τ') is) a locally p-convex algebra for some $p \in (0, 1]$, are found. It is shown that on every unital advertibly complete locally uniformly A-pseudoconvex algebra E there exists a submultiplicative semi-norm $|\,.\,|$ such that $(E, |\,.\,|)$ is a Q-algebra.

1. Introduction. 1. Let (E, τ) be a locally pseudoconvex algebra over $\mathbf{C}$ with separately continuous multiplication (in short lpca) the topology τ of which has been given by a family $\{|\,.\,|_i : i \in I\}$ of p_i-homogeneous semi-norms $|\,.\,|_i$, where $0 < p_i \leq 1$ for each $i \in I$. In particular, when $p = \inf p_i > 0$, this lpca (E, τ) is a *locally p-convex* algebra (in short lp-ca) that is, an lpca in which every $p_i = p$.

If for any $x \in E$ there is a positive number $M(x)$ such that[1]

$$\max(|xy|_i, |yx|_i) \leqslant M(x)^{p_i} |y|_i \tag{1}$$

for each $y \in E$ and $i \in I$ (here $M(x)$ depends only on x, but not on i), then an lpca (E, τ) is a *locally uniformly A-pseudoconvex* algebra (in short lu$A$-pca) and if every semi-norm $|\,.\,|_i$ in the family $\{|\,.\,|_i : i \in I\}$ is *submultiplicative*, that is,

$$|xy|_i \leqslant |x|_i |y|_i$$

for each $x, y \in E$, then an lpca (A, τ) is a *locally multiplicatively pseudoconvex* (*or locally m-pseudoconvex*) *algebra* (in short lm-pca).

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