ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 37, Number 1, 2007

PAIRS OF TOPOLOGICAL ALGEBRAS

MART ABEL AND MATI ABEL

ABSTRACT. Let (A, B) be a pair of topological algebras A and B. Conditions for A, respectively B, to be a Gelfand-Mazur algebra or an exponentially galbed algebra, if B, respectively A, is one, are given. It is shown that $\hom A$, the set of all nonzero continuous homomorphisms from A onto $\mathbf{K}$ endowed with Gelfand topology, and $\hom B$ are homeomorphic if either $\hom A$ is equicontinuous or $\hom B$ is locally equicontinuous. Topological algebras A with jointly continuous multiplication for which a) the completion $\tilde{A}$ is a Gelfand-Mazur algebra or exponentially galbed algebra or b) $\hom A$ and $\hom \tilde{A}$ are homeomorphic are described.

1. Introduction. Let A be an associative topological algebra over the field $\mathbf{K}$ (of real or complex numbers) with separately continuous multiplication (in the sequel, a topological algebra), $m(A)$ the set of such closed regular two-sided ideals of A which are maximal as left or right ideals and $\hom A$ the set of all nonzero continuous homomorphisms from A onto $\mathbf{K}$ endowed, as usual, with the topology in which a base of neighborhoods of $\varphi_0 \in \hom A$ consists of sets

$$O(\varphi_0; a_1, \dots, a_n, \varepsilon) = \bigcap_{k=1}^{n} \{\varphi \in \hom A : |(\varphi - \varphi_0)(a_k)| < \varepsilon\}$$

for some $n \in \mathbf{N}$, $\varepsilon > 0$ and $a_1, \dots, a_n \in A$. The set $\hom A$ is *equicontinuous* if, for any $\varepsilon > 0$, there is a neighborhood O of zero in A such that $|\varphi(a)| < \varepsilon$ for each $a \in O$ and $\varphi \in \hom A$ and $\hom A$ is *locally equicontinuous* if every $\varphi_0 \in \hom A$ has an equicontinuous neighborhood. It is known (see, for example, [**19**, p. 75]) that $\hom A$ is equicontinuous if A is a Q-*algebra*, that is, a topological algebra in which the set of quasi-invertible elements is open.

AMS *Mathematics Subject Classification.* Primary 46H05, Secondary 46H20.
Key words and phrases. Topological algebras, Gelfand-Mazur algebras, exponentially galbed algebras, simplicial topological algebras, Q-algebras, pairs of topological algebras, completion of topological algebras.
Research is in part supported by Estonian Science Foundation grant 6205.
Received by the editors on April 4, 2003, and in revised form on April 25, 2005.