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EXACT STRUCTURE OF POSITIVE SOLUTIONS FOR A P-LAPLACIAN PROBLEM INVOLVING SINGULAR AND SUPERLINEAR NONLINEARITIES

SHIN-HWA WANG AND TZUNG-SHIN YEH

ABSTRACT. We study the structure of positive solutions for a p-Laplacian boundary value problem involving singular and superlinear nonlinearities. We prove that there exists $\lambda^* > 0$ such that the problem has exactly two positive solutions for $0 < \lambda < \lambda^*$, exactly one positive solution for $\lambda = \lambda^*$, and no positive solution for $\lambda > \lambda^*$. More precisely, we give a complete description of the structure of the solution set. Our result partially generalizes some results of Wei [**12**].

1. Introduction. In this paper we study the structure of positive solutions $u \in C^1[-1,1] \cap C^2(-1,1)$ of the nonlinear two point boundary value problem

$$\begin{cases} (\varphi_p(u'(x)))' + \lambda \sum_{i=1}^m a_i u^{q_i} + \sum_{j=1}^n b_j u^{r_j} = 0, \quad -1 < x < 1, \\ u(-1) = u(1) = 0, \end{cases} \tag{1.1}$$

where $p > 1$, $\varphi_p(y) = |y|^{p-2}y$, $(\varphi_p(u'))'$ is the one-dimensional p-Laplacian, $\lambda > 0$ is a bifurcation parameter, and $f_\lambda = \lambda \sum_{i=1}^m a_i u^{q_i} + \sum_{j=1}^n b_j u^{r_j}$ satisfies

$$\begin{cases} -1/(p+1) \le q_1 < q_2 < \cdots < q_m < p-1 \\ \le r_1 < r_2 < \cdots < r_n, \quad m, n \ge 1, \\ q_1 < 0, \quad r_n > p-1, \quad a_i > 0 \quad \text{for } i = 1, 2, \ldots, m \\ \text{and } b_j > 0 \quad \text{for } j = 1, 2, \ldots, n, \\ \text{and (either } r_1 > p-1 \text{ or } b_1 < (p-1)((\pi/p)\csc(\pi/p))^p). \end{cases} \tag{1.2}$$

Note that, in (1.2),

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