EXPLICIT EQUATIONS OF
SOME ELLIPTIC MODULAR SURFACES

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ABSTRACT. We present explicit equations of semi-stable elliptic surfaces (i.e., having only type I_n singular fibers) which are associated to the torsion-free genus zero congruence subgroups of the modular group as classified by A. Sebbar.

1. Introduction. The purpose of this paper is to give explicit equations for the elliptic modular surfaces associated to torsion-free genus zero congruence subgroups Γ of $PSL_2(\mathbf{Z})$. We will call them *elliptic modular surfaces of genus zero* for short. By the Noether formula, the Euler number of any elliptic surface is a (nonnegative) multiple of 12. It turns out that an elliptic modular surface of genus zero has Euler number one of $12, 24, 36, 48$ and 60. Indeed, if it has an Euler number bigger than 12, then it is semi-stable and the Euler number equals the index of the corresponding group Γ in $PSL_2(\mathbf{Z})$. Sebbar's classification [17] of all such torsion free congruence subgroups Γ of genus zero implies that they have index ≤ 60 in $PSL_2(\mathbf{Z})$.

This paper can be regarded as a natural sequel to the article of Beauville [2] which deals with elliptic modular surfaces (of genus zero) having Euler number 12, and the article of Livné and Yui [12] which considered the case of elliptic modular surfaces of genus zero with Euler number 24.

We will recall the results of Beauville [2] and of Livné and Yui [12] (presenting a different approach in the latter case). Furthermore, we give explicit defining equations for the elliptic modular surfaces with Euler number one of $36, 48$ and 60.

Although our approach is different, these results could also be found using McKay and Sebbar's tables in [13], which provide one with the