ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 37, Number 2, 2007

ELLIPTIC FIBRATIONS OF SOME EXTREMAL $K3$ SURFACES

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ABSTRACT. This paper is concerned with the construction of extremal elliptic $K3$ surfaces. It gives a complete treatment of those fibrations which can be derived from rational elliptic surfaces by easy manipulations of their Weierstrass equations. In particular, this approach enables us to find explicit equations for 38 semi-stable extremal elliptic $K3$ fibrations, 32 of which are indeed defined over $\mathbf{Q}$. They are realized as pull-back of non semi-stable extremal rational elliptic surfaces via base change. This is related to the work of J. Top and N. Yui which exhibited the same procedure for the semi-stable extremal rational elliptic surfaces.

1. Introduction. The aim of this paper is to find all extremal elliptic $K3$ fibrations which can be derived from rational elliptic surfaces by direct, relatively simple manipulations of their Weierstrass equations. The main technique for this purpose will be pull-back by a base change. We only exclude the general construction involving the induced J-map of the fibration (considered as a base change generally of degree 24, conf. [**10**, Section 2]). The base changes we construct will have degree at most 8. Additionally there is another effective method if we allow the extremal $K3$ surface to have nonreduced fibres. Then we can also manipulate the Weierstrass equations by adding or transferring common factors, thus changing the shape of singular fibres rather than introducing new cusps. In total this approach will enable us to realize 201 out of the 325 configurations of singular fibres which exist for extremal elliptic $K3$ surfaces due to the classification of [**15**]. Note, however, that the configuration does in general not determine the isomorphism class.

For most of this paper, we will concentrate on the extremal elliptic $K3$ fibrations with only semi-stable fibres. The determination of the

2000 AMS *Mathematics Subject Classification.* Primary 14J27, 14J28.
This paper was partially supported by the DFG-Schwerpunktprogramm "Globale Methoden in der komplexen Geometrie."
Received by the editors on September 29, 2004, and in revised form on January 25, 2005.