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ALGEBRAIC VECTOR BUNDLES ON $\mathrm{SL}(3, \mathbf{C})$

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ABSTRACT. We show that all algebraic vector bundles on $\mathrm{SL}(3, \mathbf{C})$ are topologically trivial.

1. Introduction. There are a large number of analogies and relations between algebra and topology, cf. [**7, 8**]. For example, Serre's conjecture, cf. [**4**–**6**], was arising from analogies between projective modules and vector bundles. In many cases, topological viewpoint inspires us with several problems on algebraic vector bundles. In this paper, we deal with algebraic vector bundles over $\mathrm{SL}(3, \mathbf{C})$.

We start with Grothendieck's theorem.

Theorem 1.1 [Grothendieck **1**]. *Let G be a semi-simple simply connected affine algebraic group over an algebraically closed field. Then $K_0(G) = \mathbf{Z}$.*

From this, all algebraic vector bundles on G are stably free. Here we say that an algebraic vector bundle E is *stably free* if there exists a trivial algebraic vector bundle F such that $E \oplus F$ is also trivial. Let us consider the question whether all algebraic vector bundles over G are free.

In the case $G = \mathrm{SL}_2$, M.P. Murthy has shown the following.

Theorem 1.2 [**8**]. *Let $A = k[x, y, z, w]/(xy - zw - 1)$ be the coordinate ring of SL_2 over any field k. Then all finitely generated projective A-modules are free.*

However, in general the answer to our question is negative. In the case $G = \mathrm{SL}_4$ we have a counterexample.

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