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DERIVATIVE RELATIONSHIPS BETWEEN VOLUME AND SURFACE AREA OF COMPACT REGIONS IN $\mathbf{R}^d$

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ABSTRACT. We explore the idea that the derivative of the volume, V, of a region in $\mathbf{R}^d$ with respect to r equals its surface area, A, where $r = d(V/A)$. We show that the families of regions for which this formula for r is valid, which we call homogeneous families, include all the families of similar regions. We determine equivalent conditions for a family to be homogeneous, provide examples of homogeneous families made up of non-similar regions and offer a geometric interpretation of r in a few cases.

1. Introduction. It is well known that there exists a remarkable derivative relationship between the area A and the perimeter P of a circle, namely

$$\frac{\mathrm{d}A}{\mathrm{d}r} = P,$$

where the variable r represents the radius of the circle. It is natural to wonder whether such a derivative relationship remains valid for other familiar shapes. At first glance, though, it does not even hold for the square when r represents the side length. However, it holds when r represents half of the side length, that is, the radius of the inscribed circle.

In a similar manner, the derivative of the volume function of a sphere is equal to the surface area, that is,

$$\frac{\mathrm{d}V}{\mathrm{d}r} = A$$

and this relationship still holds for cubes if r represents the radius of the inscribed sphere.

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