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TILING THE UNIT SQUARE WITH 5 RATIONAL TRIANGLES

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ABSTRACT. There are 14 distinct ways to tile the unit square (modulo the symmetries of the square) with 5 triangles such that the 5-tiling is not a subdivision of a tiling using fewer triangles. We demonstrate how to construct infinitely many *rational* tilings in each of the 14 configurations. This stands in contrast to a long standing inability to find rational 4-tilings of the unit square in the so-called χ-configuration.

1. Introduction. Recall that a *rational triangle* is a triangle whose sides have rational length and consider the following:

Question 1. *For each $n \in \mathbf{N}$, in what ways can the unit square be tiled with n rational triangles?*

It is clear that the unit square cannot be divided into two rational triangles and, in [**7**], Guy similarly disposes of the case $n = 3$. Guy goes on to prove that there are essentially four distinct ways to tile the square with four triangles and along with Bremner [**2, 3**] proved that at least three of them admit rational tilings. The goal of this paper is to solve the $n = 5$ case. Before moving on however, it is worth noting that the remaining $n = 4$ case is the subject of problem D19 in [**8**] and can be articulated as:

Question 2. *Is there a point on the interior of the unit square that is a rational distance to each of the four corners?*

More formally, and in keeping with the language established by Guy, we say that a *proper n-tiling*, or simply an *n-tiling*, is a set of n triangles

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