ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 32, Number 3, Fall 2002

ITERATIVE APPROXIMATION OF SOLUTIONS TO NONLINEAR EQUATIONS OF ϕ-STRONGLY ACCRETIVE OPERATORS IN BANACH SPACES

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ABSTRACT. Suppose that X is an arbitrary Banach space and $T : X \to X$ is a uniformly continuous ϕ-strongly accretive operator. It is proved that, for a given $f \in X$, the Ishikawa iteration method with errors converges strongly to the solutions of the equations $f = Tx$ and $f = x + Tx$ under suitable conditions. Related results deal with the iterative approximation of fixed points of ϕ-strongly pseudocontractive operators. Our results generalize, improve and unify the corresponding results in [**2**]–[**11**], [**13**]–[**16**], [**19**], [**20**], [**23**]–[**26**] and [**28**].

1. Introduction. Let X be an arbitrary Banach space with norm $\|\cdot\|$ and dual X^* and J denote the normalized duality map from X into 2^{X^*} given by

$$J_X = \{f^* \in X^* : \|f^*\|^2 = \|x\|^2 = \operatorname{Re}\langle x, f^*\rangle\},$$

where $\langle\cdot,\cdot\rangle$ stands for the generalized duality pairing between X and X^*. It is well known that, if X^* is convex, then J is single-valued. In the sequel we shall denote the single-valued duality mapping by j.

An operator T with domain $D(T)$ and range $R(T)$ in X is called strongly accretive if there exists a constant $k > 0$ such that for all $x, y \in D(T)$, there exists $j(x-y) \in J(x-y)$ satisfying

$$\operatorname{Re}\langle Tx - Ty, j(x-y)\rangle \geq k\|x-y\|^2. \tag{1.1}$$

Without loss of generality we may assume $k \in (0,1)$. If $k = 0$ in (1.1), then T is called accretive. Furthermore, T is called ϕ-strongly accretive

1991 AMS *Mathematics Subject Classification.* 47H17, 47H10, 47H05.
Key words and phrases. ϕ-strongly accretive operators, ϕ-strongly pseudocontractive operators, Mann iteration process with errors, Ishikawa iteration process with errors, fixed points, Banach spaces.
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Received by the editors on May 19, 2000, and in revised form on August 11, 2000.