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LINEAR RATIONAL INTERPOLATION AND ITS APPLICATION IN APPROXIMATION AND BOUNDARY VALUE PROBLEMS

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ABSTRACT. We consider the case that a function with large gradients in the interior of an interval has to be approximated over this interval or that the pseudospectral method is used to compute a similar solution of an ordinary boundary value problem. In both cases we assume that the function has minimal continuity properties but can be evaluated anywhere in the given interval. The key idea is then to attach poles to the polynomial interpolant, respectively solution of the collocation problem to obtain a special rational function with poles whose location has been optimized suitably. In the first case, the max norm of the error is minimized while in the second, the same norm is minimized of the residual of the given differential equation. The algorithms are presented and discussed. Their effectiveness is demonstrated with numerical results.

1. Introduction. In this paper we address two problems which are not necessarily related but for both of which we propose in principle the same basic approach. The first problem is that of interpolating a given continuous function f between $N+1$ distinct points $x_0, x_1, \dots, x_N$ in an interval $[a,b]$. We can choose $[a,b] = [-1,1]$ without loss of generality. The second problem is that of solving on the same interval the boundary value problem (BVP)

$$\begin{gathered} u''(x) + p(x)u'(x) + q(x)u(x) = f(x) \\ u(-1) = u_l, \quad u(1) = u_r, \end{gathered}$$

where all arising functions belong to $C^\infty[-1,1]$ and where u_l and u_r are given real numbers. For more details on both problems, see [**16, 17**].

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