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FROM NON-HERMITIAN OSCILLATOR-LIKE OPERATORS TO FREUD POLYNOMIALS AND SOME CONSEQUENCES

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ABSTRACT. Non-Hermitian quantum Hamiltonians dealing with oscillator-like interactions are discussed when realized in terms of creation and annihilation operators that are no longer adjoint to each other. Specific differential realizations are exploited and lead to real spectra and typical eigenfunctions including (unexpected) Freud orthogonal polynomials. Hermiticity is finally revisited with respect to new scalar products of specific Hilbert spaces.

Subnormality [**14**] of linear operators is a relatively recent mathematical property. An operator S in H is said to be *subnormal* if it has a normal extension N (recall that N is normal on K including H if $\|Nf\| = \|N^\dagger f\|$ for $f \in D(N) = D(N^\dagger)$). This subnormality has not yet been exploited in physics up to several remarks on the (bosonic) *creation operator* (denoted by $a^\dagger$) in the context of (one-dimensional) quantum harmonic oscillators. Such an operator, considered as the *best representative* of subnormal (unbounded) operators, generates with its companion, a, i.e., the (bosonic) annihilation operator, the Lie-Heisenberg commutation relations [**4**] associated to this study. Acting on a Hilbert space, currently called the Fock-Bargmann space [**3**], characterized by orthonormalized state-vectors $|n>$, $n = 0, 1, 2, \dots$, the corresponding Hamiltonian

$$H_{H.O.} = a^\dagger a + \frac{1}{2} = \frac{1}{2}(a^\dagger a + a a^\dagger) \equiv \frac{1}{2}\{a^\dagger, a\} \tag{1}$$

is directly expressed in terms of these operators and appears trivially as a *self-adjoint* operator (with respect to the well-known scalar product of the above-mentioned Hilbert space) admitting trivially a *real* spectrum as expected.

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