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ALGEBRA OF DIFFERENTIAL FORMS WITH EXTERIOR DIFFERENTIAL $d^3 = 0$ IN DIMENSIONS ONE AND TWO

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ABSTRACT. In this paper, we construct the algebra of differential forms with exterior differential satisfying $d^3 = 0$ over an associative algebra with one and n generators satisfying quadratic relations. Supposing $d^2 \neq 0$, we introduce the second order differentials d^2x^i. We also assume that the homomorphism defining a first order differential calculus is linear in variables, and that there are no relations between the terms $(dx^i)^2$ and d^2x^j. A graded q-differential algebra with $d^3 = 0$ is constructed by means of the Wess-Zumino method. The commutation relations between generators x^i, dx^j, d^2x^k of the algebra of differential forms in pairs and themselves are found. In the case of the algebra with n generators, the commutation relations between noncommutataive derivatives ∂_i and generators d^2x^j also are found, and the consistency conditions are described.

1. Introduction. An idea to generalize the classical exterior differential calculus with $d^2 = 0$ to the case $d^N = 0$, $N > 2$, arises in a recent series of papers [**2–4, 6**], where the different approaches to this idea are developed, and these generalizations have been proposed and studied. In the paper [**5**] such a generalization is provided by the notion of graded q-differential algebra which is, according to the definition given in [**2**], an associative unital **N**-graded algebra endowed with a linear endomorphism d (q-differential) of degree 1 satisfying $d^N = 0$ and the graded q-Leibniz rule

$$d(\omega\tau) = d(\omega)\tau + q^{\mathrm{gr}(\omega)}\omega\, d(\tau), \tag{1}$$

where ω, τ are arbitrary elements of the algebra; $\mathrm{gr}(\omega)$ is the grade of an element ω; q is a primitive cubic root of unity.

In the paper [**5**], a q-differential calculus with $d^3 = 0$ is constructed on a classical smooth n-dimensional manifold. We construct the q-differential calculus on an associative algebra generated by one variable

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