ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 32, Number 2, Summer 2002

A MULTI-INDEX BOREL-DZRBASHJAN TRANSFORM

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ABSTRACT. An integral transform involving a Fox's H-function is introduced. This integral transform is closely related to a multi-index analogue of the classical Mittag-Leffler function. Along with the basic operational and mapping properties of this transform, the new results presented here include complex and real inversion formulas and a convolution theorem.

1. Introduction. The role of the *Laplace transform*:

$$\mathcal{L}\{f(z);s\} = \int_0^\infty \exp(-sz) f(z)\, dz \tag{1}$$

in the operational calculus, and its use in various problems of applied analysis, engineering and other fields are well-known. The success of the Laplace transform motivates the search for other more general transforms of similar type. As an integral transform of resembling type, one can mention the *Borel-Dzrbashjan transform*, studied initially by Dzrbashjan [**5**], and later by Dimovski and Kiryakova [**3**]:

$$\mathcal{B}_{\rho,\mu}\{f(z);s\} = \rho s^{\mu\rho-1} \int_0^\infty \exp(-s^\rho z^\rho) z^{\mu\rho-1} f(z)\, dz, \tag{2}$$
$$\rho > 0, \quad \mu > 0.$$

Another transform of the same type that is related to the Bessel differential operator is the well-known *Meijer transform*:

$$\mathcal{K}_\nu\{f(z);s\} = \int_0^\infty \sqrt{sz} K_\nu(sz) f(z)\, dz, \tag{3}$$

2000 AMS *Mathematics subject classification.* 44A15, 44A30, 26A33, 33E12, 33C60.

Keywords and phrases. Laplace-type integral transform, Mittag-Leffler function, H-transform, operators of fractional integration and differentiation, Borel-Dzrbashian transform.

Received by the editors on November 30, 2000, and in revised form on May 8, 2001.

The second author partially supported by NSF-Bulg. Ministry of Education and Science under Grant MM 708.

Research of the third author supported by the Kuwait University Research Administration under Grant SM 05/00.