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PARABOLIC WAVELET TRANSFORMS AND LEBESGUE SPACES OF PARABOLIC POTENTIALS

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ABSTRACT. Parabolic wavelet transforms associated with heat operators $-\mathcal{D}_x + \partial/\partial t$ and $I - \mathcal{D}_x + \partial/\partial_t$ in $\mathbf{R}^{n+1}$ are introduced. A Calderón-type reproducing formula for functions $f \in L^p(\mathbf{R}^{n+1})$ is proven. By making use of these transforms, new explicit inversion formulas for the Jones-Sampson parabolic potentials are obtained, and characterization of the corresponding anisotropic Lebesgue spaces is given.

1. Introduction and main results. Continuous wavelet transforms

$$Wf(x,a) = \frac{1}{a^n} \int_{\mathbf{R}^n} f(y) w\left(\frac{|x-y|}{a}\right) dy,$$

$x \in \mathbf{R}^n$, $a > 0$, $\int_{\mathbf{R}^n} w(|y|)\, dy = 0$, play an important role in analysis and have many applications, (see, e.g., [**7**–**9, 13, 15, 16, 22, 23, 26**] and references therein). Due to the formula

$$\int_0^\infty Wf(x,a) \frac{da}{a^{1+\alpha}} = c(-\mathcal{D})^{\alpha/2} f(x), \quad c = c(\alpha, w),\ \alpha \in \mathbf{C}, \tag{1.1}$$

which gives an integral representation of powers of the Laplacian $\mathcal{D} = \partial^2/\partial x_1^2 + \cdots + \partial^2/\partial x_n^2$, the wavelet transforms Wf are applicable to a variety of problems in PDE, integral geometry and function theory [**13, 15, 16, 22, 24**]. The formula (1.1) can be justified in the framework of the L^p-theory [**22**].

In the present paper, we introduce anisotropic analogues of Wf which enable one to obtain wavelet-type representations, like (1.1), of powers

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