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CONGRUENCES FOR THE COEFFICIENTS WITHIN A GENERALIZED FACTORIAL POLYNOMIAL

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1. Introduction. The Stirling numbers of first order, denoted by $s(n,k)$ can be defined for $n>0$ as the coefficient of x^k in the expansion of the rising factorial polynomial

$$x(x+1)\cdots(x+n-1)=\sum_{k=1}^{n} s(n,k)x^k.$$

The many varied properties of this class of numbers have been extensively studied, see, for example, [**2**]. Yet, in spite of this, congruences for the Stirling numbers $s(n,k)$ are apparently not well known. A few congruences for prime moduli can be found in [**2**] and other texts, but it has only been in recent times that certain papers have appeared dealing specifically with the problem of Stirling number congruences (see [**1**], [**3**]). Of these papers, the one of most interest to us here is due to Howard, who found congruences (mod p) for $s(n,k)$ and the associated Stirling numbers. We briefly list some of the main congruences as follows:

$$s(p,k)\equiv 0 \pmod p \quad \text{for } 2\le k\le p-1 \tag{1}$$

$$s(p-1,k)\equiv 1 \pmod p \quad \text{for } 1\le k\le p-1 \tag{2}$$

$$s(p-2,k)\equiv (2^{p-k-1}-1) \pmod p \quad \text{for } 0\le k\le p-2 \tag{3}$$

$$s(hp+m,k)\equiv \sum_{i=0}^{h}\binom{h}{i}(-1)^{h-i}s(m,k-h-i(p-1)) \pmod p \quad \text{for } 0\le m<p. \tag{4}$$

As an application of the above results, a complete examination of the congruences (mod p) for $s(n,k)$, where $n\ge p$, was given in [**3**] for the special cases $p=2,3$ and 5. In this paper we propose to extend

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