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THE ISOTOPY CLASSIFICATION OF AFFINE QUARTIC CURVES

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To the memory of D.A. Gudkov

ABSTRACT. In this paper we obtain the isotopy classification of affine quartic curves, which contains 647 classes, and the topological classification of pairs ($\mathbf{R}^2$, *quartic curve*), which contains 516 classes (see Theorem 7). We also present the isotopy classification of real projective quartic curves, which contains 66 classes.

We prove that each of these classifications is equivalent to the classification of all real (affine or projective) quartic curves having only singular points, if any, of types A_1, A_1^*, D_4 or X_9 (see Theorems 5 and 6 and Corollaries 6.1–6.4).

1. Introduction. One of the most important problems in the topology of real algebraic curves and surfaces is the problem of their classification. Any such classification is based on an equivalence relation for the set of varieties being considered. The quotient set with respect to the equivalence relation is called a *classification*. There are a number of different approaches to the subject. One can consider classification of varieties either of fixed degree or of fixed dimension or of both. One can also consider only nonsingular varieties, or varieties with other prescribed algebraic or topological properties. Historically, the first and most basic approach is to classify real algebraic varieties up to affine equivalences in $\mathbf{R}^n$, respectively, projective equivalence in $\mathbf{R}P^n$. Two affine (projective) varieties are called *affine* (*projective*) *equivalent* provided there exists a nondegenerate affine (projective) linear transformation that carries one variety to the other. The affine (projective) linear transformation does not change the degree of a variety. Thus

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