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A WEAK EFFECTIVE ROTH'S THEOREM OVER FUNCTION FIELDS

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1. Introduction. The correspondence between Diophantine approximation and Nevanlinna theory, observed by Osgood and Vojta [**2**], [**7**], has motivated many recent works. Furthermore, the Diophantine approximation over function fields also has recently attracted attention because of its correspondence to Nevanlinna theory with moving targets. In [**8**], Julie Wang obtained an effective version of Roth's theorem over function fields by adapting the method of Steinmetz in proving Nevanlinna's conjecture with slowly moving targets in Nevanlinna theory. We note that the Thue-Siegel-Roth theorem over function fields was proved by Uchiyama [**6**] in 1961, with a proof similar to the one for number fields, hence is ineffective. To state Wang's result we recall some definitions. Let C be an irreducible nonsingular algebraic curve of genus g over an algebraically closed field k of characteristic zero. Let K be the function field of C. For a nonzero element $f \in K$, we define the height as $h(f) = \sum_{P\in C} -\min\{0, v_P(f)\}$, where $v_P(f)$ is the order of f at the point P of C. Let t be a nonconstant function in K; we denote by, for every $y \in K$, $y' = (dy/dt)$. Julie Wang's result is stated as follows:

Theorem [8]. *Let S be a finite set of points in C. Suppose that $t, a_1, \dots, a_q$ are S-units and that f is a nonzero element of K. Let $L(r)$ be the vector space over k spanned by $a_1^{n_1} \dots a_q^{n_1}$ with $n_1, \dots, n_q \geq 0$ and $n_1 + \dots + n_q = r$. Let $\beta_1, \dots, \beta_n$ be a base of $L(r)$ and $b_1, \dots, b_m$ a base of $L(r+1)$. If $f\beta_1, \dots, f\beta_n, \beta_1, \dots, b_m$ are linearly independent*

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