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REDUCTIONS OF A GENERALIZED INCOMPLETE GAMMA FUNCTION, RELATED KAMPÉ DE FÉRIET FUNCTIONS, AND INCOMPLETE WEBER INTEGRALS

ALLEN R. MILLER

ABSTRACT. We derive several reduction formulas for specializations of a certain generalized incomplete gamma function and its associated Kampé de Fériet function $F^{0:2;1}_{2:0;0}[x, y]$. Reductions of specializations of incomplete Weber integrals of modified Bessel functions and related functions $F^{1:0;0}_{1:0;1}[x, y]$, $F^{0:1;0}_{1:0;0}[x, y]$ heretofore also unavailable are deduced.

1. Introduction. One of a class of generalized incomplete gamma functions may be defined by

$$\Gamma(\nu, x; z) \equiv \int_x^\infty t^{\nu-1} e^{-t-z/t}\, dt, \tag{1.1}$$

where the parameters ν, x and argument z are arbitrary complex numbers. When the argument z vanishes, $\Gamma(\nu, x; z)$ reduces to the ordinary incomplete gamma function $\Gamma(\nu, x)$ of classical analysis. Although several authors (see [**1**], [**2**], [**6**], [**8**]) have studied this particular generalization, it appears that the last word concerning properties of the latter has not been said. Indeed, Veling [**8**] has recently recorded without explicit proof the reduction formula for nonnegative integers n

$$\Gamma(n, z; z^2) = z^n [K_n(2z) + e^{-2z} U_n(2z)], \tag{1.2a}$$

where

$$\begin{aligned} U_n(z) \equiv{}& K_n(z)\Big[I_0(z) + 2\sum_{j=0}^{n-2} I_{j+1}(z)\Big] \\ &- (-1)^n I_n(z)\Big[K_0(z) - 2\sum_{j=0}^{n-2}(-1)^j K_{j+1}(z)\Big], \end{aligned} \tag{1.2b}$$

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